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CHANGES IN APPARENT SIZE OF GIANT STARS WITH WAVELENGTH DUE TO ELECTRON-HYDROGEN COLLISIONS

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ABSTRACT

Interferometric measurements of stellar sizes in frequency bands ranging from the near-infrared to longer wavelengths give different results. Various explanations have been proposed to account for these variations in apparent size with wavelength, but none have been entirely consistent. We propose that thermal ionization in the stellar atmosphere and resulting opacity, primarily due to free-free electron-hydrogen collisions, play a significant role. Such an opacity has a quadratic dependence on photon wavelength and produces variations in the opacity of the atmosphere with wavelength, consistent with pertinent measurements. This may be particularly important for Mira-type stars, and two examples, o Ceti and W Hya, are analyzed as examples. For stars that are much smaller or with more concentrated mass, it is not likely to be significant.

Subject headings: stars: atmospheres — stars: fundamental parameters — stars: variables: other — techniques: interferometric

1. INTRODUCTION

Measurements of stellar size at optical, near-IR, and mid-IR wavelengths show substantial differences, which have generally been attributed to surrounding molecules and dust (e.g., Weiner 2004). It is proposed here that for large old stars that are not very massive, much of this difference at longer wavelengths ($\geq 2\ \mu$m) is due to continuum radiation associated with electron-hydrogen collisions, also known as bremsstrahlung, in the atmospheres of stars. The opacity per unit distance increases roughly as the square of the wavelength, and thus the atmosphere is opaque for longer wavelengths at larger radii, enlarging the apparent stellar size with increasing wavelength. The enlarged apparent size due to ionization of stellar atmospheres has already been recognized in the microwave region, where an increase of measured radius in the range 50%–100% has been noted (Reid & Menten 1997). At wavelengths down into the infrared this effect has been largely neglected, but it can make an important contribution to apparent stellar size.

2. SOURCES OF CONTINUUM EMISSION

Continuum radiation is produced by electron-electron, electron-ion, electron-hydrogen atom, and electron-hydrogen molecule collisions. In each case, the intensity of radiation for an optically thin material, and hence the opacity per unit distance, is closely proportional to the square of the wavelength and, if the fractional ionization is constant, to the square of the atmospheric density. As a rough approximation consider the atmosphere close to a star of constant fractional ionization, constant temperature, and primarily composed of hydrogen atoms but varying in density in accordance with hydrostatic equilibrium. The variation in gas density with distance from a star is approximately

$$n = n_0 \exp\left[-\frac{GMm_{\rm H}}{kT}\left(\frac{1}{R_0} - \frac{1}{R}\right)\right] \quad (1)$$

$$= n_0 \exp\left[-S\left(1 - \frac{R_0}{R}\right)\right], \quad (2)$$

where $S \equiv (GMm_{\rm H}/kT)(1/R_0)$. Here, n_0 is the atomic density at R_0, the stellar radius as measured at optical wavelengths ($\lambda \leq 1\ \mu$m), n is the density at a somewhat increased radius R, G is the gravitational constant, M is the stellar mass, $m_{\rm H}$ is the hydrogen atom mass, and T is the temperature, which as an approximation is assumed constant over small changes of the radius, R.

For a star such as the Sun, $S \approx 4.0 \times 10^3$, leading to the density changing by a factor of 10 for a fractional change in R of only 6×10^{-4}. Hence, particle-particle collisions cannot change the size noticeably with increase in wavelength from the optical to any infrared wavelength. However, for Mira-type stars with masses comparable to that of the Sun, temperatures of about 3000 K and radii approximately 500 times larger than the Sun, $S \approx 15.4$. In this case, for a change in density of a factor of 10, $R \approx 1.18R_0$. Hence, if this type of continuum radiation is important, the ratio of apparent radius at 10 μm to that at 1 μm wavelength would be 1.18, which is in rough agreement with observed results for o Ceti. With these simplifying assumptions, the apparent radius compared with that at 1 μm would increase by a factor of approximately 1.3, 1.81, and 2.49 for wavelengths 100 μm, 1 mm, and 1 cm, respectively. The importance of such radiation compared to other types of opacity is discussed in detail below.

A detailed and specific examination of stellar characteristics, atmospheric composition, density, and variation in temperature is now considered. Dust or solid particles do not form close to the star, but if not carefully taken into account, they can affect the apparent stellar size by absorbing stellar radiation or by contributing radiation at some distance from the star. Spectral lines and continuum emission due to collisions contribute opacity at all distances from the star and can affect its apparent size. Spectral lines are generally most important at the shorter wavelengths, while continuum absorption due to collisions is most important at longer wavelengths.

First, atmospheric opacity and apparent stellar size due to electron-hydrogen atom collisions are discussed and calculated, since at wavelengths greater than about 1.6 μm, this appears to be the primary source of atmospheric opacity (other than possible dust). After numerical examination of this type of opacity, other types are considered. These other types are shown, in most situations, to be substantially smaller and can be neglected.

TABLE 1

Atmospheric Density as a Function of Distance from a Mira Star such as o Ceti, Assuming a Stable Atmosphere in Equilibrium

R/R_0	n/n_0 for Constant T = 2700 K	$T = 2700(R_0/R)^{1/2}$ (K)	n/n_0 for $T = 2700(R_0/R)^{1/2}$
1.00	1.00	2700	1.00
1.05	3.28×10^{-1}	2635	3.20×10^{-1}
1.10	1.18×10^{-1}	2574	1.08×10^{-1}
1.15	4.65×10^{-2}	2518	3.80×10^{-2}
1.20	1.97×10^{-2}	2465	1.40×10^{-2}
1.30	4.26×10^{-3}	2368	2.13×10^{-3}
1.40	1.13×10^{-3}	2282	3.70×10^{-4}
1.50	3.57×10^{-4}	2204	7.17×10^{-5}
1.60	1.29×10^{-4}	2135	1.54×10^{-5}
1.70	5.19×10^{-5}	2071	3.58×10^{-6}
1.80	2.30×10^{-5}	2012	8.93×10^{-7}
1.90	1.11×10^{-5}	1959	2.40×10^{-7}
2.00	5.69×10^{-6}	1909	6.78×10^{-8}

Notes.—The stellar mass is assumed to be that of the Sun, the radius, R_0, 400 times larger than that of the Sun, and the stellar temperature 2700 K. R is the radial distance, n the density at this distance, and n_0 the density at the stellar surface.

TABLE 2

Relative Abundance and Fractional Ionization of Elements that Contribute the Majority of Electrons in Stellar Atmospheres due to Thermal Ionization

Element	Abundance Relative to H	Ionization Potential (eV)	T (K)	n	n_i/n
Fe	3.98×10^{-5}	7.87	3000	10^{14}	7.5×10^{-2}
	3.98×10^{-5}	7.87	3000	10^{12}	5.3×10^{-1}
	3.98×10^{-5}	7.87	2500	10^{14}	3.2×10^{-3}
	3.98×10^{-5}	7.87	2500	10^{12}	3.2×10^{-2}
Al	2.46×10^{-6}	5.99	3000	10^{14}	1.00
	2.46×10^{-6}	5.99	3000	10^{12}	1.00
	2.46×10^{-6}	5.99	2500	10^{14}	0.63
	2.46×10^{-6}	5.99	2500	10^{12}	1.00
	2.46×10^{-6}	5.99	2000	10^{14}	2.6×10^{-2}
	2.46×10^{-6}	5.99	2000	10^{12}	2.3×10^{-1}
Na	1.78×10^{-6}	5.14	3000	10^{14}	1.00
	1.78×10^{-6}	5.14	3000	10^{12}	1.00
	1.78×10^{-6}	5.14	2500	10^{14}	1.00
	1.78×10^{-6}	5.14	2500	10^{12}	1.00
	1.78×10^{-6}	5.14	2000	10^{14}	0.62
	1.78×10^{-6}	5.14	2000	10^{12}	0.94
K	8.91×10^{-8}	4.34	2000–3000	10^{12}–10^{14}	1.00

Note.—The last three columns contain the fractional ionization n_i/n as a function of T and n, the density of H cm^{-3}.

For a very simple model we assume a quiet stellar atmosphere, where shocks or gas flow do not substantially affect the density, and also that there is a fixed relative atomic abundance in an atmosphere dominated by hydrogen atoms. We further assume a temperature distribution as a function of radius from the star of the form

$$T = T_0(R_0/R)^{\alpha}, \quad (3)$$

where T_0 is the temperature at R_0, the optical stellar radius. For a fixed temperature, the coefficient α would be zero, or α may be taken as 1/2 for a temperature decreasing as $(1/R)^{1/2}$. This assumes a constant total outward flux proportional to T^4R^2. For equilibrium in a stable and stationary atmosphere, the precise relation for atomic density variation with radial distance is

$$n = n_0\left(\frac{R_0}{R}\right)^{2-\alpha} \exp\left\{\frac{-GMm_{\rm H}}{kT(1-\alpha)R_0}\left[1-\left(\frac{R_0}{R}\right)^{1-\alpha}\right]\right\}, \quad (4)$$

where n_0 is the atomic density at R_0. Except for the slowly varying factor of $(R_0/R)^2$, this is identical to equation (2) when the temperature is constant ($\alpha = 0$). Convection and other turbulent effects likely disturb the equilibrium of the atmosphere, affecting the density. However, at higher altitudes, where the opacity begins to fall below unity, these effects are generally not very important (Brown et al. 1989).

The electron density is mostly produced by thermal ionization of the chemical elements, and for cool old stars, the ionized elements are largely metals with low ionization potentials, such as Na and Al. The density of ions n_i for a given chemical element of ionization potential V is given by the Saha equation,

$$\frac{n_i n_e}{n_0} = \frac{(2\pi m_e kT)^{3/2}}{h^3}\frac{2u_i(T)}{u_0(T)} \exp\left(-\frac{V}{kT}\right), \quad (5)$$

where n_0 is the density of unionized atoms, n_e is the density of electrons, m_e is the electron mass, and h is Planck's constant. The partition functions for ionized and neutral states are u_i and u_0, respectively. Chemical equilibrium, assumed in equation (5), is justified because of the rather high atom density of gas near the stellar radius. As a specific example, Table 1 gives the relative density as a function of distance from a Mira-type star such as o Ceti and an atmosphere of fixed temperature, as well as one varying in temperature proportional to $R^{-1/2}$.

It might be thought that gas outflow from the star, such as a Mira, would appreciably affect the atomic or molecular density. This is not the case. A typical average outflow might be $10^{-6}\ M_{\odot}$ yr^{-1} at a velocity of 20 km s^{-1}, which would result in a density of about 1.2×10^9 atoms cm^{-3} at a radius of 500 $R_{\odot}$, considerably below the expected atmospheric density of perhaps 10^{14}–10^{15} cm^{-3}. Since the stable atmospheric density decreases roughly exponentially with distance, while an outflow decreases more slowly, proportional to R^{-2}, such an outflow can make a significant contribution to density much further away from the star. However, this would occur at radii greater than about 1.8 times the optical stellar radius, larger than those generally considered here.

Table 2 gives the assumed abundance of elements relative to H, which are those for the Sun (Allen 1985), and the fractional ionization for several different elements as a function of temperature and density. Na is the largest contributor of free electrons for relatively low temperatures, since it has the lowest ionization potential with the exception of K, which is substantially less abundant.

3. H SPECIES ABUNDANCES AND OPACITIES DUE TO ELECTRON COLLISIONS WITH THEM

Free electrons, for which densities can be obtained from Table 2, produce continuum radiation or absorption by collision with themselves, with positive ions, with H atoms or molecules, or with negative ions such as H$^-$. We next consider the relative abundance of these constituents and the resulting continuum radiation or absorption.

TABLE 3

Values of N_H^2/N_{H_2} as a Function of Temperature, from which Relative Abundances of H and H_2 Can Be Obtained

T (K)	N_H^2/N_{H_2}
1500	6.61×10^9
1800	2.33×10^{12}
2000	4.40×10^{13}
2250	8.36×10^{14}
2500	8.89×10^{15}
2700	4.30×10^{16}

The abundance of H compared with H_2 is given by

$$\frac{N_H^2}{N_{H_2}} = \frac{(2\pi m_{H_2} kT)^{3/2}}{h^3} \exp\left(-\frac{D}{kT}\right) \frac{U_H^2}{Q}, \tag{6}$$

where N_H and N_{H_2} represent the densities of H and H_2, respectively, m_{H_2} is the H_2 molecular mass, D is the disassociation energy of H_2, which is 4.477 eV, and U_H is the partition function for H, which is 2. The partition function for H_2 is Q, primarily due to multiple rotational states, which for a kT much larger than the rotational constant is approximately $Q = kT/hcB$, where c is the speed of light and B is the rotational constant in cm^{-1}, equal to 60.81 cm^{-1}.

The values of N_H^2/N_{H_2} are given in Table 3 as a function of temperature. The hydrogen density at the surface of a Mira-type star is estimated to be in the range of 10^{14}–10^{16} cm^{-3}, which is similar to figures assumed by others such as Reid & Menten (1997) and Luttermoser et al. (1994). Table 1 gives its decrease as a function of distance from the star and the temperature. From the numbers in Table 3 one can show that for hydrogen densities of 10^{14} to 10^{16} cm^{-3}, the fractional atomic population ranges from 1.00 to 0.74 at a temperature of 2700 K. Hence, close to the star, where the temperature is high, or further away, where the temperature is lower but so is the density, $N_H \gg N_{H_2}$, i.e., most of molecular hydrogen is dissociated throughout the stellar atmosphere.

The electron density in stellar atmospheres can be estimated from Tables 1 and 2. At temperatures that apply to the atmospheres of Mira-type stars, Na is a primary contributor of electrons. Potassium is rather completely ionized, but has an abundance 20 times less than Na. Al contributes electrons at the higher temperatures, and near the stellar surface adds to the Na electron density by a factor of as much as about 1.4. This does not increase electron density by any large amount, but in some cases is significant. As an approximation, we shall primarily consider only the Na contribution, which means that $n_e \approx n_i$ in equation (5).

The density of H^- in thermal equilibrium is

$$n_{H^-} = \frac{n_H n_e h^3}{4(2\pi m_e kT)^{3/2}} \exp\left(\frac{\chi}{kT}\right), \tag{7}$$

where χ is the binding energy of the electron in H^-, which is 0.754 eV. This makes the ratio of H^- to n_e

$$\frac{n_{H^-}}{n_e} = \frac{3.27 \times 10^{-21}}{(T/1000)^{3/2}} n_H \exp\left(\frac{8.704}{T/1000}\right). \tag{8}$$

For $T = 2500$ K, equation (8) gives $(n_{H^-}/n_e) = 2.7 \times 10^{-20} n_H$, which for any values of n_H pertinent to stellar atmospheres shows that $n_{H^-} \ll n_e$. The density of H^- may not be determined by thermal equilibrium alone, since processes such as photodetachment of the electron from H^- and H_2 formation also occur. However, these tend to decrease the density of H^-, and in any case, its density is not large enough to be important when compared to electron densities. With the above abundances of H, H^-, electrons, and ions, it turns out that electron-hydrogen collisions are clearly the largest source of continuum radiation or absorption at the longer wavelengths.

The absorption coefficient due to electron-hydrogen atom collisions has been given by Dalgarno & Lane (1966) and simplified somewhat by Reid & Menten (1997), whose results are used. The absorption coefficient is

$$a_\nu = (kT n_e n_H A)/\nu^2, \tag{9}$$

where ν is the frequency, and A is given at longer wavelengths by

$$A = 3.376 \times 10^3 - 2.149 \times 10^3 (T/1000) + 6.646 \times 10^2 (T/1000)^2 - 7.853 \times 10^1 (T/1000)^3. \tag{10}$$

This value of A is accurate to better than 1% at frequencies as high as 10^{13} Hz and to 10% for those as high as 10^{14} Hz, or a wavelength of about 3 μm. The value of A is 10^3 at a temperature of 2300 K, and changes only about $\mp 25\%$ for $T = 1500$ and 3000 K. At $\lambda = 10$ μm or $\nu = 3 \times 10^{13}$ Hz, if $n_e/n_H = 1.8 \times 10^{-6}$, as given by the fraction of Na atoms, then

$$a_\nu = 2.76 \times 10^{-43} (T/1000) n_H^2 \text{ cm}^{-1}. \tag{11}$$

A common definition of the radius of a star is the distance from the stellar center at which a line of sight past the star encounters an optical depth of unity. Calculation of optical depth involves a detailed model of the atmosphere, since the absorption per unit distance varies along the line of sight. For simplicity, we assume here that the stellar radius is that at which the atmospheric absorption per unit distance produces an optical depth of unity for a path length of one-half the radius, $L = 0.5R$. For such a path length, $\pm 0.25R$ from the point at which the line of sight is perpendicular to the stellar radius, the distance from the stellar center increases only about $0.03R$, decreasing $(n_H)^2$ by a factor of about 0.40 so that the absorption coefficient does not decrease greatly. By comparison, a distance of R, or $\pm 0.5R$, from the point where the line of sight is perpendicular to the stellar radius would decrease the absorption coefficient by a factor of about 0.04 at the more extreme distance from the star, so the effective path length cannot be as long as R. With this approximation, the optical depth for $\lambda = 10$ μm is then, for an old star of radius $R = 500\ R_\odot$,

$$a_\nu L = 5 \times 10^{-30} (T/1000) n_H^2. \tag{12}$$

Near the star, where $T \approx 2500$ K, equation (12) gives an optical depth of unity at $\lambda = 10$ μm if $N_H = 2.8 \times 10^{14}$ cm^{-3}, which is a reasonable estimate of the hydrogen density near the star. For a wavelength $\lambda = 1$ μm, at which an optical depth of unity should essentially define the stellar radius, N_H would need to be 2.8×10^{15} cm^{-3} for $a_\nu L = 1$. This makes it clear that electron-hydrogen atom collisions are an important source of opacity that affects the apparent stellar size. Electron-hydrogen molecule collisions contribute a smaller opacity because opacities per electron collision with H and H_2 are comparable, while, as shown above, the abundance of H is substantially greater than that of H_2.

Consider now the electron-ion collision process, in which electrons collide with Na^+ or any other ion. The absorption coefficient due to such collisions is (Reid & Menten 1997)

$$a_{\rm ion} = \frac{4e^6}{3c}\left(\frac{2\pi}{3}\right)^{1/2}\left(\frac{1}{m_e kT}\right)^{3/2}\frac{n_e n_i}{\nu^2}g_{\rm ff}, \tag{13}$$

where n_e and n_i are the densities of electrons and ions, respectively, and $g_{\rm ff}$ is the Gaunt factor, which varies from approximately 1 to approximately 4 as the wavelength varies from 1 to 10 μm. This gives a value at $\lambda = 10$ μm of

$$a_{\rm ion} \approx \left[\frac{7.5\times10^{-34}}{(T/1000)^{3/2}}\right] n_e n_i. \tag{14}$$

Since n_e and n_i depend primarily on the ionization of Na and are hence smaller than the hydrogen density by about a factor of about 2×10^{-6}, comparison of equations (11) and (14) shows that the opacity due to electron-ion or electron-electron collisions is substantially smaller than that due to electron-hydrogen atom collisions.

An important contribution to stellar atmospheric opacity at wavelengths in the near-IR and shorter is photon absorption that detaches an electron from H^-. Wavelengths must be shorter than 1.6 μm to cause such a detachment, so this process is unimportant at longer wavelengths. All transitions to a bound state due to a single photon are forbidden in H^-, since the binding of the extra electron is due to correlations of the electrons' motions rather than coulomb attraction alone. This means that in order for the extra electron to remain bound, both electrons must be simultaneously excited by a two-photon transition, an effect that can be neglected here.

The absorption cross section for H^- peaks at a wavelength of about 0.85 μm with a value 3.8×10^{-17} μm^2 and decreases to essentially zero for wavelengths greater than 1.65 μm. At 1.5 μm its value is about 8×10^{-18}, and hence, with a density of 10^5 corresponding to an H density of about 10^{15} cm^{-3} and a temperature of 2500 K, the absorption coefficient would be 8×10^{-13} cm^{-1}. For a distance $\frac{1}{2}\times500$ $R_\odot$ near an old star, the opacity αL would then be quite large, approximately 14. H^- quantum absorption and photodetachment can thus be an important contributor to opacity in stellar atmospheres for wavelengths shorter than 1.65 μm and may contribute to variations in apparent stellar size at these short wavelengths. Details of its density and opacity calculation are not further considered here. Present emphasis is on wavelengths greater than 1.65 μm, where electron collisions with H, discussed above, appear to dominate the continuum opacity.

4. PHOTOIONIZATION

Photoionization from ultraviolet light in the star's blackbody radiation is another candidate source of free electrons. For cool old stars, however, this effect does not play a significant role in generating a free electron population. Photoionization is the overwhelmingly dominant effect around hotter O-type stars whose spectra contain more power in the ultraviolet. These stars generate a region of fully ionized hydrogen surrounding the star, a Strömgren sphere. The old stars to be considered here are, however, much too cool to create enough high-energy photons to make this process significant.

The mean free path (MFP) of a photon is inversely proportional to the density of absorbing atoms and their individual cross sections. Thus the MFP, ℓ, for photons through hydrogen is

$$\ell = \frac{1}{\sigma_{\rm H} n_{\rm H}} \tag{15}$$

where $\sigma_{\rm H} = 6.3\times10^{-18}$ cm^2 is the cross section of hydrogen at the ionizing energy, and $n_{\rm H}$ is the density of hydrogen in atoms per unit volume. Assume the density of hydrogen atoms in the stellar atmosphere to be $n_{\rm H} \approx 10^{15}$ cm^{-3}. At this density the MFP is $\ell = 1.6\times10^2$ cm. This means, assuming the fraction of ionized to neutral hydrogen is small, that the photoionized electrons will be confined to a region very near the star's surface. Even if 90% of the hydrogen is ionized and the actual density of neutral hydrogen is an order of magnitude smaller, the photons will still not penetrate any significant distance into the stellar atmosphere.

To estimate the actual fraction of ionized hydrogen, we assume that the total ionization rate of hydrogen is equal to the recombination rate between free electrons and hydrogen ions. The ionization rate is given by the rate at which photons at or above the ionization energy are emitted, since each of these photons will ionize an atom. This is approximately

$$N_\gamma = \frac{15(\beta E_\gamma)^3}{\pi^4 \exp(\beta E_\gamma)}\frac{A\sigma T^4}{E_\gamma}, \tag{16}$$

with $\beta \equiv (kT)^{-1}$. The first fraction in this equation is an approximate expression for the portion of energy in a thermal distribution of photons integrated from an energy of $E_\gamma = 13.6$ eV to infinity. This approximate expression is valid as long as $E_\gamma \gg kT$. The latter fraction is the total power emitted as a function of temperature divided by the energy per photon. We assume all photons to have the same energy, E_γ. This is approximately true, since the number of photons at a given energy drops exponentially with increasing energy. Here also A is the surface area of the photosphere, σ is the Stefan-Boltzmann constant, and T is the temperature. Expressing σ in terms of fundamental constants and simplifying the expression for N_γ yields

$$N_\gamma = \frac{2\pi A E_\gamma^2}{\beta \exp(\beta E_\gamma) h^3 c^2}. \tag{17}$$

The recombination rate, due primarily to collisions and then photon emission, is given by

$$N_r = r n_e n_p V \approx r n_e^2 A \ell_i, \tag{18}$$

where n_e and n_p are the electron and hydrogen ion concentrations, respectively, r is a coefficient giving the efficiency of recombination, and V is the total volume of the ionized shell from the stellar surface out to where the gas is once again unionized. The term V is approximated here as $A\ell_i$, the surface area times the shell thickness, ℓ_i. Setting $N_\gamma = N_r$ and solving for n_e using equations (17) and (18) yields

$$n_e = \sqrt{\frac{2\pi E_\gamma^2}{\exp(\beta E_\gamma) h^3 c^2 \beta r \ell_i}}. \tag{19}$$

Consider a star with a surface temperature of $T = 2500$ K. At this temperature $\beta = 4.64$ eV^{-1} and r is approximately

6×10^{-13} cm^3 s^{-1} (Allen 1985). Taking $\ell_i = \ell$ gives an electron or hydrogen ion density of 4.0×10^3 cm^{-3}, which is indeed a small fraction of the neutral hydrogen density. This means that our initial calculation for the MFP remains valid for cool, old stars even in the presence of photoionization. The photoionization generates electrons only in a region very close to the stellar surface and, according to this calculation, ionizes only a small fraction of the hydrogen atoms.

Easily ionized metals in the stellar atmosphere, such as sodium, also do not contribute significantly to the free electron density as the result of photoionization. The cross section of sodium is $\sigma_{\rm Na} = 1.3 \times 10^{-19}$ cm^2 (Isenberg et al. 1985). The relative abundance of sodium is around 1.78×10^{-6}, giving a MFP of $\ell = 4.3 \times 10^9$ cm, a small fraction of the stellar radius, which is approximately 3×10^{13} cm. Although many more photons are emitted at sodium's ionization energy of 5.4 eV or above, the ionization is still not a significant effect. The recombination efficiency, r, of sodium is 4×10^{-16} cm^3 s^{-1}. This is somewhat less than for hydrogen, but the electron density depends only on the square root of r, which gives an electron density of hundreds cm^{-3}, still a very small quantity compared to that produced by thermal ionization.

Two photon ionization processes are also small compared to thermal effects. The dominant two-photon process is excitation to the first excited state, the Lyα transition, followed by ionization. Here we can make the "on-the-spot" approximation that if one atom decays and emits a Lyα photon, it will be quickly reabsorbed by another atom nearby. Since there is no other significant decay channel from the first excited state, atoms in the first excited state are essentially unable to decay, so there may be a substantial fraction of hydrogen in the first excited state. Ionization from the first excited state still requires 10.2 eV, and the proportion of photons at this energy is relatively small. The photoionization cross section of hydrogen in the first excited state is also higher than in the ground state. Thus, a photon at 10.2 eV will not penetrate a significant distance into the stellar atmosphere in the presence of even a modest fraction of excited atoms.

Other transitions also occur but with decreasing significance. Excitations to levels higher than the Lyα require higher energies that rapidly approach 13.6 eV, giving results similar to the neutral hydrogen calculations.

5. EFFECT OF THE ELECTRON-HYDROGEN COLLISIONAL CONTINUUM ON APPARENT RADIUS FOR SPECIFIC STELLAR MODELS

The contribution of continuum radiation produced by electron-hydrogen atom collisions to an apparent stellar radius was perhaps first emphasized by Reid & Menten (1997) in discussion of the microwave size of W Hya. It has also been discussed by Weiner (2002) in his thesis. However, neither Reid & Menten (1997) nor Weiner (2002) made a point of its contribution to apparent stellar size at infrared wavelengths. Measurements of W Hya by Reid & Menten (1997) at a wavelength of 1.36 cm give an apparent size of 80 mas, 1.7–2.0 times larger than the estimated optical size of the star, and a temperature at that radius of 1630 K. From a calculation of electron density needed to produce the observed opacity, they obtained a hydrogen density at this distance from the star of approximately 1.5×10^{12} cm^{-3}. Their model of the stellar atmosphere gives a temperature of approximately 2400 K and a density of 2×10^{14} cm^{-3} at the stellar radius, or 431 $R_\odot$. A mass of 0.5 $M_\odot$ was assumed. For such conditions and at this chosen stellar radius, equations (4), (5), and (9) give an optical depth of unity due to electron-hydrogen collisional continuum for a wavelength of about 5 μm, so the optical radius or the density may be somewhat less than their model, although not far different. The ratio of apparent size between 10 and 1 μm wavelengths due to electron-hydrogen collisions would be approximately 1.20.

The apparent size of o Ceti at 11 μm wavelength is approximately 48 mas (Weiner et al . 2000) and at near-IR wavelengths (2.2 μm), approximately 34 mas (Tuthill et al. 1999). Particularly in the near-IR, this apparent radius can be much affected by dust or molecular absorption, so it is not very accurately determined. The ratio of apparent radii at 11 and 2.2 μm from the electron-hydrogen continuum radiation would be expected to be approximately 1.11, assuming the reasonable estimates of a mass $M_\odot$, radius 400 $R_\odot$, and temperature at the stellar surface of 2700 K. Clearly, other effects are present but a significant portion of the change in apparent size is likely due to the effects of the electron-hydrogen continuum. The presence of dust and molecules clearly give additional complications to near-IR measurements. The 11 μm measurements of size are not as affected by dust surrounding the star. It has been suggested that water vapor contributes significantly to the star's apparent size at 11 μm (Weiner 2004; Ohnaka et al. 2006). On the other hand, the measured 11 μm spectrum of o Ceti does not agree with the water vapor lines expected, if in fact water is a dominant contributor to the larger apparent diameter at 11 μm wavelengths. Hence, it appears logical that much of the enlarged apparent size is due to the continuum absorption discussed here. These effects should be more obvious when the apparent size at 11 μm compared to millimeter wavelength measurements is examined, since spectral features and dust are expected to play less of a role. Lack of data in the millimeter range prevents such comparisons at this time, and millimeter size measurements would be interesting and important.

Water vapor may in fact have a substantial effect on the apparent size of o Ceti at shorter wavelengths, as suggested by Weiner (2004) and Ohnaka et al. (2006). The mystery of the lack of water lines in the 11 μm region sufficiently strong to have a major effect on the apparent size of o Ceti is, however, eliminated by the continuum absorption intensity at this longer wavelength due to electron-hydrogen collisions, which can produce the larger size.

Another type of stellar example in which the apparent size varies substantially with wavelength is α Ori. In this case, the apparent size at 11 μm is 54.7 mas, while at visible wavelengths, when visibility data are fitted to a uniform disk model, it has been found in the range 42–52 mas (Cheng et al. 1986; Buscher et al. 1990; Wilson et al. 1992), with an average of perhaps 48 mas, close to Michelson's original measurement (Michelson & Pease 1921). Thus, the ratio of apparent sizes for 11 μm and $\sim$1 μm is approximately 1.14. Because of the continuum radiation, making reasonable estimates of the mass of α Ori as 10 $M_\odot$, a temperature of 3600 K and the distance as 130 pc so that its radius is 700 $R_\odot$, the ratio of apparent radii at 11 and 1 μm would be only 1.05. This ratio is smaller than that for o Ceti, primarily because the estimated mass is larger by a factor of 10. The fact that it is not as large as the measured difference in apparent size may well be associated with limb darkening at the shorter wavelengths or it could be partly related to differences in the actual decrease of atmospheric density with distance, which is assumed here to correspond to a static equilibrium atmosphere. There may be dynamic motions in the atmosphere that modify the density change with distance from the star. If the mass of α Ori were 5 instead of 10 $M_\odot$, the calculated ratio of sizes at 11 and 1 μm would increase from 1.05 to 1.10.

In addition to the continuum opacity explaining at least some of the difference in apparent size as a function of wavelength, its use can provide information on the stellar mass, size, and atmospheric characteristics, particularly at the longer wavelengths where effects of dust or molecular and atomic line absorption are less important than for short wavelengths. Already, Reid & Menten (1997) have used the measurements of W Hya at centimeter wavelengths to obtain information on its atmospheric density. For o Ceti, the 11 μm apparent size measurement can give information on the atmospheric density at its apparent radius. For an optical depth near unity and if the stellar distance is 80 pc, the density of atoms must be close to 3.0×10^{14} cm^{-3} at the apparent stellar radius of 24 mas. At the 2.2 μm stellar radius, less than 24 mas by the factor 1/1.11, the density would hence be approximately 4.0×10^{15} cm^{-3}.

6. CONCLUSIONS

It is clear that at wavelengths longer than 1.6 μm collisions between electrons and hydrogen in stellar atmospheres produce a continuum radiation and opacity that play an important, often dominant, role in the apparent stellar size. There are, however, other phenomena such as spectral lines and dust that play an important role, particularly in the near-IR and visible regions and where additional complex theoretical calculations are needed to understand the apparent size of stars, especially old stars surrounded by dust and gas. At the longer wavelengths, infrared and millimeter, where the electron-hydrogen continuum is dominant, measurement of its effect on apparent size can give useful information about stellar parameters such as mass, distance, and atmospheric density.

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REFERENCES

Allen, C. W. 1985, Astrophysical Quantities (3rd reprint ed.; London: Athlone)
Brown, J. A., Johnson, H. R., Alexander, D. R., Cutright, L. C., & Sharp, C. M. 1989, ApJS, 71, 623
Buscher, D. F., Haniff, C. A., Baldwin, J. E., & Warner, P. J. 1990, MNRAS, 245, 7P
Cheng, A. Y. S., Hege, E. K., Hubbard, E. N., Goldberg, L., Strittmatter, P. A., & Cocke, W. J. 1986, ApJ, 309, 737
Dalgarno, A., & Lane, L. F. 1966, ApJ, 145, 623
Isenberg, M. E., Carter, S. L., Kelly, H. P., & Salomonson, S. 1985, Phys. Rev. A, 32, 1472
Luttermoser, D. G., Johnson, H. R., & Eaton, J. 1994, ApJ, 422, 351
Michelson, A. A., & Pease, F. G. 1921, ApJ, 53, 249
Reid, M. J., & Menten, K. M. 1997, ApJ, 476, 327
Tuthill, P. G., Haniff, C. A., & Baldwin, J. E. 1999, MNRAS, 306, 353
Weiner, J. 2002, Ph.D. thesis, Univ. of California, Berkley
———. 2004, ApJ, 611, L37
Weiner, J., Danchi, W. C., Hale, D. D. S., McMahon, J., Townes, C. H., Monnier, J. D., & Tuthill, P. G. 2000, ApJ, 544, 1097
Wilson, R. W., Baldwin, J. E., Buscher, D. F., & Warner, P. J. 1992, MNRAS, 257, 369