### Physical Constants

<table>
<thead>
<tr>
<th>Name</th>
<th>Symbol</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number (\pi)</td>
<td>(\pi)</td>
<td>3.14159265358979323846</td>
<td></td>
</tr>
<tr>
<td>Number (e)</td>
<td>(e)</td>
<td>2.71828182845904523536</td>
<td></td>
</tr>
<tr>
<td>Euler’s constant</td>
<td>(\gamma)</td>
<td>(\lim_{n \to \infty} \left( \sum_{k=1}^{n} \frac{1}{k} - \ln(n) \right) = 0.5772156649)</td>
<td>()</td>
</tr>
<tr>
<td>Elementary charge</td>
<td>(e)</td>
<td>1.60217733 (\cdot) (10^{-19})</td>
<td>C</td>
</tr>
<tr>
<td>Gravitational constant</td>
<td>(G, \kappa)</td>
<td>6.67259 (\cdot) (10^{-11})</td>
<td>m(^3)kg(^{-1})s(^{-2})</td>
</tr>
<tr>
<td>Fine-structure constant</td>
<td>(\alpha = \frac{e^2}{2\hbar c \varepsilon_0})</td>
<td>(\approx \frac{1}{137})</td>
<td></td>
</tr>
<tr>
<td>Speed of light in vacuum</td>
<td>(c)</td>
<td>2.99792458 (\cdot) (10^{8})</td>
<td>m/s (def)</td>
</tr>
<tr>
<td>Permittivity of the vacuum</td>
<td>(\varepsilon_0)</td>
<td>8.854187 (\cdot) (10^{-12})</td>
<td>F/m</td>
</tr>
<tr>
<td>Permeability of the vacuum</td>
<td>(\mu_0)</td>
<td>4(\pi) (\cdot) (10^{-7})</td>
<td>H/m</td>
</tr>
<tr>
<td>((4\pi\varepsilon_0))(^{-1})</td>
<td></td>
<td>8.9876 (\cdot) (10^9)</td>
<td>Nm(^2)C(^{-2})</td>
</tr>
<tr>
<td>Planck’s constant</td>
<td>(h)</td>
<td>6.6260755 (\cdot) (10^{-34})</td>
<td>Js</td>
</tr>
<tr>
<td>Dirac’s constant</td>
<td>(\hbar = h/2\pi)</td>
<td>1.0545727 (\cdot) (10^{-34})</td>
<td>Js</td>
</tr>
<tr>
<td>Bohr magneton</td>
<td>(\mu_B = \frac{e h}{2m_e})</td>
<td>9.2741 (\cdot) (10^{-24})</td>
<td>Am(^2)</td>
</tr>
<tr>
<td>Bohr radius</td>
<td>(a_0)</td>
<td>0.52918</td>
<td>A</td>
</tr>
<tr>
<td>Rydberg’s constant</td>
<td>(R_y)</td>
<td>13.595</td>
<td>eV</td>
</tr>
<tr>
<td>Proton Compton wavelength</td>
<td>(\lambda_{cp} = \frac{h}{m_pc})</td>
<td>1.3214 (\cdot) (10^{-15})</td>
<td>m</td>
</tr>
<tr>
<td>Reduced mass of the H-atom</td>
<td>(\mu_H)</td>
<td>9.1045755 (\cdot) (10^{-31})</td>
<td>kg</td>
</tr>
<tr>
<td>Stefan-Boltzmann’s constant</td>
<td>(\sigma)</td>
<td>5.67032 (\cdot) (10^{-8})</td>
<td>Wm(^{-2})K(^{-4})</td>
</tr>
<tr>
<td>Wien’s constant</td>
<td>(k_W)</td>
<td>2.8978 (\cdot) (10^{-3})</td>
<td>mK</td>
</tr>
<tr>
<td>Molar gas constant</td>
<td>(R)</td>
<td>8.31441</td>
<td>J(\cdot)mol(^{-1})K(^{-1})</td>
</tr>
<tr>
<td>Avogadro’s constant</td>
<td>(N_A)</td>
<td>6.0221367 (\cdot) (10^{23})</td>
<td>mol(^{-1})</td>
</tr>
<tr>
<td>Boltzmann’s constant</td>
<td>(k = R/N_A)</td>
<td>1.380658 (\cdot) (10^{-23})</td>
<td>J/K</td>
</tr>
<tr>
<td>Electron mass</td>
<td>(m_e)</td>
<td>9.1093897 (\cdot) (10^{-31})</td>
<td>kg</td>
</tr>
<tr>
<td>Proton mass</td>
<td>(m_p)</td>
<td>1.6726231 (\cdot) (10^{-27})</td>
<td>kg</td>
</tr>
<tr>
<td>Neutron mass</td>
<td>(m_n)</td>
<td>1.674954 (\cdot) (10^{-27})</td>
<td>kg</td>
</tr>
<tr>
<td>Elementary mass unit</td>
<td>(m_u = \frac{1}{12}m(\overset{12}{\overset{6}{C}}))</td>
<td>1.6605656 (\cdot) (10^{-27})</td>
<td>kg</td>
</tr>
<tr>
<td>Nuclear magneton</td>
<td>(\mu_N)</td>
<td>5.0508 (\cdot) (10^{-27})</td>
<td>J/T</td>
</tr>
<tr>
<td>Diameter of the Sun</td>
<td>(D_\odot)</td>
<td>1392 (\cdot) (10^6)</td>
<td>m</td>
</tr>
<tr>
<td>Mass of the Sun</td>
<td>(M_\odot)</td>
<td>1.989 (\cdot) (10^{30})</td>
<td>kg</td>
</tr>
<tr>
<td>Rotational period of the Sun</td>
<td>(T_\odot)</td>
<td>25.38</td>
<td>days</td>
</tr>
<tr>
<td>Radius of Earth</td>
<td>(R_A)</td>
<td>6.378 (\cdot) (10^6)</td>
<td>m</td>
</tr>
<tr>
<td>Mass of Earth</td>
<td>(M_A)</td>
<td>5.976 (\cdot) (10^{24})</td>
<td>kg</td>
</tr>
<tr>
<td>Rotational period of Earth</td>
<td>(T_A)</td>
<td>23.96</td>
<td>hours</td>
</tr>
<tr>
<td>Earth orbital period</td>
<td>Tropical year</td>
<td>365.24219879</td>
<td>days</td>
</tr>
<tr>
<td>Astronomical unit</td>
<td>AU</td>
<td>1.4959787066 (\cdot) (10^{11})</td>
<td>m</td>
</tr>
<tr>
<td>Light year</td>
<td>lj</td>
<td>9.4605 (\cdot) (10^{15})</td>
<td>m</td>
</tr>
<tr>
<td>Parsec</td>
<td>pc</td>
<td>3.0857 (\cdot) (10^{16})</td>
<td>m</td>
</tr>
<tr>
<td>Hubble constant</td>
<td>(H)</td>
<td>(\approx (75 \pm 25))</td>
<td>km(\cdot)s(^{-1})(\cdot)Mpc(^{-1})</td>
</tr>
</tbody>
</table>
The Theory of Special Relativity

The Lorentz Transformations

The Lorentz transformation equations linking space and time coordinates \((x, y, z, t)\) and \((x', y', z', t')\) of the same event measured from \(S\) and \(S'\) are

\[
x' = \frac{x - ut}{\sqrt{1 - u^2/c^2}} \quad (1)
\]

\[
y' = y \quad (2)
\]

\[
z' = z \quad (3)
\]

\[
t' = \frac{t - ux/c^2}{\sqrt{1 - u^2/c^2}} \quad (4)
\]

The ubiquitous factor of

\[
\gamma \equiv \frac{1}{\sqrt{1 - u^2/c^2}} \quad (5)
\]

is called The Lorentz factor, and may be used to estimate the importance of relativistic effects. The inverse transformations are

\[
x = \frac{x + ut}{\sqrt{1 - u^2/c^2}} \quad (6)
\]

\[
y = y \quad (7)
\]

\[
z = z \quad (8)
\]

\[
t = \frac{t + ux/c^2}{\sqrt{1 - u^2/c^2}} \quad (9)
\]

Time and Space In Special Relativity

The time interval \(\Delta t \equiv t_2 - t_1\) between the same two flashes measured by a clock in frame \(S\) is

\[
t_2 - t_1 = (t'_2 - t'_1) + (x'_2 - x'_1)u/c^2 \quad (10)
\]

or

\[
\Delta t = \gamma \Delta t' \quad (11)
\]

which is also

\[
\Delta t_{\text{moving}} = \gamma \Delta t_{\text{rest}} \quad (12)
\]

This equation shows the effect of time dilation on a moving clock.

Proper Length and length contraction

The length \(L = x_2 - x_1\) measured in the \(S\) frame may be found from

\[
x'_2 - x'_1 = \gamma [(x_2 - x_1) - u(t_2 - t_1)] \quad (13)
\]

or

\[
L' = \gamma L \quad (14)
\]

Because the rod is at rest relative to \(S'\), \(L'\) will be called \(L_{\text{rest}}\). Similarly, because the rod is moving relative to \(S\), \(L\) will be called \(L_{\text{moving}}\), and we get

\[
L_{\text{moving}} = \frac{L_{\text{rest}}}{\gamma} \quad (15)
\]

This equation shows the effect of length contraction on a moving rod.

The Relativistic Doppler Shift

The relativistic Doppler shift is defined as

\[
\nu_{\text{obs}} = \nu_{\text{rest}} \sqrt{1 - v_r/c} \quad (16)
\]

A redshift parameter \(z\) is used to describe the change in wavelength; it is defined as

\[
z \equiv \frac{\lambda_{\text{obs}} - \lambda_{\text{rest}}}{\lambda_{\text{rest}}} = \frac{\Delta \lambda}{\lambda_{\text{rest}}} \quad (17)
\]

The observed wavelength \(\lambda_{\text{obs}}\) is obtained from Equation 16 and \(c = \lambda v\)

\[
\lambda_{\text{obs}} = \lambda_{\text{rest}} \sqrt{\frac{1 + v_r/c}{1 - v_r/c}} \quad (18)
\]

and the redshift parameter becomes

\[
z = \sqrt{\frac{1 + v_r/c}{1 - v_r/c}} - 1 \quad (19)
\]

In general,

\[
z + 1 = \frac{\Delta t_{\text{obs}}}{\Delta t_{\text{rest}}} \quad (20)
\]

The Relativistic Velocity Transformation

The equations describing the relativistic transformation of velocities may be easily found from the Lorentz transformation equations by writing them as differentials. Then dividing the \(dx', dy',\) and \(dz'\) equations by the \(dt'\) equation gives the relativistic velocity transformations

\[
x'_2 - x'_1 = \gamma [(x_2 - x_1) - u(t_2 - t_1)] 
\]

\[
y' = y
\]

\[
z' = z
\]

\[
t' = \frac{t - ux/c^2}{\sqrt{1 - u^2/c^2}}
\]
\[ v'_x = \frac{v_x - u}{1 - uv_x/c^2} \]  
(21)
\[ v'_y = \frac{v_y \sqrt{1 - u^2/c^2}}{1 - uv_x/c^2} \]  
(22)
\[ v'_z = \frac{v_z \sqrt{1 - u^2/c^2}}{1 - uv_x/c^2} \]  
(23)

the inverse transformations are given by
\[ v_x = \frac{v'_x + u}{1 + uv_x/c^2} \]  
(24)
\[ v_y = \frac{v'_y \sqrt{1 - u^2/c^2}}{1 + uv_x/c^2} \]  
(25)
\[ v_z = \frac{v'_z \sqrt{1 - u^2/c^2}}{1 + uv_x/c^2} \]  
(26)

Relativistic Momentum and Energy
The relativistic momentum vector is defined as
\[ \mathbf{p} = \gamma m \mathbf{v} \]  
(27)
The relativistic kinetic energy is
\[ K = mc^2 (\gamma - 1) \]  
(28)
The total relativistic energy \( E \) is given by
\[ E = \gamma mc^2 \]  
(29)
and
\[ E_{\text{rest}} = mc^2 \]  
(30)

A useful expression relating a particle’s total energy \( E \), the magnitude of its momentum \( p \), and its rest energy \( mc^2 \) is
\[ E^2 = p^2 c^2 + m^2 c^4 \]  
(31)

General Relativity
\[ F = G \frac{Mm}{r^2} \]  
(32)
The Principal of Equivalence
\[ ma_g = G \frac{Mm}{r^2} \]  
gravitational force (33)
\[ ma_e = \frac{qQ}{4\pi\epsilon_0 r^2} \]  
electric force (34)

separating inertial mass and gravitationally mass yields
\[ a_g = \frac{G m_g m_i}{r^2} \]  
(35)
\[ a_e = \frac{1}{4\pi\epsilon_0 r^2} \]  
(36)

The principal of equivalence: All local, freely falling, non-rotating laboratories are fully equivalent for the performance of all physical experiments.

radius of curvature
\[ r_c = \frac{c^2}{L} \]  
(37)
gravitational redshift and time dilation
\[ \frac{\Delta \nu}{\nu_0} = \frac{v}{c} = \frac{gh}{c^2} \]  
(38)
integrating this equation gives
\[ \int_{\nu_0}^{\nu} \frac{d\nu}{\nu} \sim - \int_{r_0}^{r} \frac{GM}{r^2 c^2} dr \]  
(39)
integrating this equation and using a taylor expansion we get
\[ \frac{\nu_\infty}{\nu_0} = (1 - 2GM/r_0 c^2)^{1/2} \]  
(40)
\[ \Delta t = \frac{1}{\nu} \]  
(41)
for a weak field
\[ \frac{\Delta t_0}{\Delta t_\infty} \sim 1 - \frac{GM}{r_0 c^2} \]  
(42)

Intervals and Geodesics
\[ G = -\frac{8\pi G}{c^4} T \]  
(43)
proper time
\[ \Delta \tau = \frac{\Delta s}{c} \]  
(44)
proper distance
\[ \Delta \mathcal{L} = \sqrt{-(\Delta s)^2} \]  
(45)

space-time interval
\[(\Delta s)^2 = [c(\Delta t_{ba})]^2 - (\Delta x_{ba})^2 - (\Delta y_{ba})^2 - (\Delta z_{ba})^2 \]

(46)

if \((\Delta s)^2 < 0\) then the interval is spacelike, if \(\Delta s)^2 > 0\) then the interval is timelike, and if \(\Delta s)^2 = 0\) then the interval is lightlike or null

\[(\Delta s)^2 = \begin{cases} < 0 & \text{the interval is spacelike} \\ = 0 & \text{the interval is lightlike} \\ > 0 & \text{the interval is timelike} \end{cases} \]

(47)

\[d\mathcal{L} = \sqrt{-(ds)^2} = \frac{dr}{\sqrt{1 - \frac{2GM}{rc^2}}} \]

(48)

\[d\tau \frac{ds}{c} = dt \sqrt{1 - \frac{2GM}{rc^2}} \]

(49)

The Schwarzschild Metric

\[(ds)^2 = (cdt\sqrt{1 - \frac{2GM}{rc^2}})^2 - \left(\frac{dr}{\sqrt{1 - \frac{2GM}{rc^2}}}\right)^2 - (rd\theta)^2 - (r \sin \theta d\phi)^2 \]